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AN INTRODUCTION TO VORTEX BREAKDOWN AND VORTEX CORE BURSTING

by

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National Aeronautical Establishment

OTTAWA

MARCH 1958

AERONAUTICAL NOTE

NAE-AN-28

NRC NO. 34256

85-4904/

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AN INTRODUCTION TO VORTEX BREAKDOWN AND VORTEX CORE BURSTING

INTRODUCTION À LA RUPTURE ET À L'ÉCLATEMENT DU NOYAU DES VORTEX

by/par

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OTTAWA
MARCH 1985

AERONAUTICAL NOTE
NAE-AN-28
NRC NO. 24336

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SUMMARY

This report is an introduction to the phenomenon of vortex breakdown, also know as vortex core bursting. The first part of the report presents some important elements of basic vortex theory. The second part reviews the research that has been done on vortex breakdown, including an overview of the historical development of vortex breakdown research and a summary of the current state of knowledge on the subject.

RÉSUMÉ

Le présent rapport est une introduction au phénomène de rupture des vortex, phénomène que l'on appelle également éclatement du noyau des vortex. La première partie du rapport présente quelques éléments importants de la théorie de base des vortex. La seconde partie passe en revue la recherche qui a été faite dans le domaine de la rupture des vortex. Un aperçu général de l'évolution historique de la recherche se rapportant à la rupture des vortex ainsi qu'un résumé de l'état actuel des connaissances en la matière sont présentés.

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AN INTRODUCTION TO VORTEX BREAKDOWN AND VORTEX CORE BURSTING

1.0 INTRODUCTION

This report is an introduction to the subject of vortex breakdown, a phenomenon also known as vortex core bursting. There are two parts to this report: the first part presents some key elements of basic vortex theory in an attempt to explain what vortices are, how they are modeled and how they are generated by aircraft wings; the second part defines vortex breakdown and then presents a review of the research that has been published on the subject.

In this report, the various topics are treated in a simplified manner. Very little of the extensive mathematical theory on vortex motion appears here. Consequently, the purposes of this report are to familiarize the reader with the basic concepts and terminology relating to vortices and to indicate the key references where further details can be found. In addition, some attempt is made to trace the historical development of vortex breakdown research and to summarize the current state of knowledge on the subject.

2.0 ELEMENTS OF BASIC VORTEX THEORY

2.1 Mathematical Formulation of Fluid Vortices

The following overview is based on the treatment of vortex theory by Milne-Thomson (1952).

The basic definition of vorticity is given by

$$\vec{\zeta} = \vec{\nabla} \times \vec{q} \tag{1}$$

where $\vec{\zeta}$ is the vorticity and $\vec{q}$ the fluid velocity at a point (x,y,z). Closely related to the vorticity is the circulation which is defined by

$$\Gamma = \int_c \vec{q} \cdot d\vec{\ell} \tag{2}$$

where Γ is the circulation (scalar quantity) around a closed path c and $d\ell$ is an elemental vector tangent to the curve c at all points. The relationship between ζ and Γ is given by Stoke's Theorem:

$$\Gamma = \iint_s \vec{\zeta} \times \vec{n}\, ds \tag{3}$$

where the surface integral is computed over any regular surface (s) bounded by the curve c and ds is an elemental area with unit outward normal $\vec{n}$.

A number of valuable insights into vortex motion can be obtained through consideration of rectilinear or two-dimensional vortices. Figure 1 shows a rectilinear vortex for which it is assumed that the vorticity has a uniform strength ω over the core region. The direction of the vortex vector is defined to be perpendicular to the plane of rotation as given by the right hand rule. Outside of the vortex core (radius a), the fluid vorticity is zero. The circulation can be easily calculated for the circular contours 1 and 2 from Equation (2):

$$\Gamma_1 = (2\pi r')v' \tag{4}$$

$$\Gamma_2 = (2\pi r)v \tag{5}$$

Equation (3) provides an alternative method of calculation for the respective circulations. We find:

$$\Gamma_1 = \omega(\pi r'^2) \tag{6}$$

$$\Gamma_2 = \omega(\pi a^2) \tag{7}$$

Equating (4) with (6), and (5) with (7) provides the relationship between the vorticity and the velocity for both the core

$$v' = \frac{1}{2}\ \omega r' \tag{8}$$

and for external regions

$$v = \frac{1}{2}\ \frac{\omega a^2}{r} \tag{9}$$

The velocity field defined by (8) and (9) is referred to as the induced velocity, and it is sketched in Figure 2. The term "induced velocity" is somewhat misleading because ". . . $\vec{\zeta}$ and $\vec{q}$ *occur together* but neither can properly be said to *cause* the other". (Milne-Thomson, 1952, p. 164) Note that if the induced velocity were the only velocity present in this example, then the streamlines would be concentric circles centered at the origin of the vortex core. Consequently, the fluid pressure must be a minimum at the origin of the vortex core in order to provide the force required to make the trajectories of the fluid particles lie along circular arcs.

The following five theorems define further properties of vortex motion:

(i) Kelvin's Circulation Theorem states: "When the external forces are conservative and derived from a one-valued potential, the circulation in any circuit which moves with the fluid (i.e. which always consists of the same fluid particles) is independent of the time." (Milne-Thomson, 1952, p. 163).

(ii) The First Theorem of Helmholtz described the general motion of a fluid particle. Consider an infinitesimal fluid particle centered at position (x,y,z), with velocity components (u,v,w) at this point. The theorem states that the fluid particle moves like a rigid body with translational velocity (u,v,w) and rotational velocity $1/2\,\vec{\zeta}$ and that superposed on these motions is a deformation velocity caused by the shearing action on the particle. Hence, it the fluid particle were suddenly frozen, it would begin to rotate with an angular velocity of one-half the fluid vorticity at the point.

(iii) A vortex filament, which is also called a vortex tube, is defined to be a stream tube containing vorticity which is oriented along the streamlines. The Second Theorem of Helmholtz states that for a vortex filament, the product of the magnitude of the vorticity and the cross-sectional area remains a constant. This product is called the intensity of the filament. This theorem is a statement that vorticity is conserved in a vortex filament; hence, vortex filaments cannot end in the interior of the fluid and must therefore form closed rings or terminate on the boundary.

(iv) The Third Theorem of Helmholtz states that, "the fluid which form a vortex tube continues to form a vortex tube". (Milne-Thomson, 1952, p. 166.)

(v) The Fourth Theorem of Helmholtz states that, "the intensity of a vortex tube remains constant as the tube moves about". (Milne-Thomson, 1952, p. 166.)

The foregoing discussion provides a brief review of the basic properties of fluid vortices. Theoretically, any inviscid, incompressible flow problem involving vorticity can be solved from the governing equations (Widnall, 1975) of Conservation of Mass:

$$\vec{\nabla} \cdot \vec{q} = 0 \tag{10}$$

and the Dynamic Equation of Vorticity:

$$\left(\frac{\partial}{\partial t} + \vec{q} \cdot \vec{\nabla}\right)\vec{\zeta} \equiv \frac{D\vec{\zeta}}{Dt} = (\vec{\zeta} \cdot \vec{\nabla})\vec{q} \tag{11}$$

These two equations must be supplemented by the definition of vorticity, which is given by Equation (1). As is common in fluid mechanical problems, these differential equations are nonlinear and cannot be solved analytically. Therefore, recourse must be made to approximate methods, a number of which will be mentioned in the specific application of solving the problem of vortex breakdown.

2.2 Aerodynamic Vortices

The following discussion follows conventional wing theory and reviews some key features of vortex generation by aircraft.

The generation of lift is directly related to the presence of circulation around an airfoil or wing. The Kutta-Joukowski Theorem relates lift to circulation for two-dimensional airfoils:

$$L_b = \rho v \Gamma \tag{12}$$

where L_b is the lift per unit span, ρ is the fluid density, v is the fluid speed and Γ is the circulation around the airfoil as defined in Equations (2) and (3). This result can be extended to finite aspect ratio wings according to the well established lifting line or lifting surface theories. In particular, for the lifting line, we have:

$$L = \rho v \int_{-b/2}^{+b/2} \Gamma(y)\, dy \tag{13}$$

where L is the total lift on the wing, b is the span of the wing, and the circulation $\Gamma(y)$ is now a function of the spanwise co-ordinate y. The problem of determining $\Gamma(y)$ given the shape of the wing, and the inverse problem of determining the shape of the wing given $\Gamma(y)$ comprise a large portion of wing theory. This theory has been treated in considerable detail in a very large number of references (for example, Milne-Thomson, 1952) and will not be discussed further in this report.

A lifting wing causes a lateral flow around the wing tips because of the higher pressure on the bottom or pressure side of the wing. This lateral flow is schematically shown in Figure 3. A discontinuity is created when the upper and lower air streams combine at the trailing edge of the wing because of the opposing lateral motions. Note that the flow *speed* is the same for both air streams because of Bernoulli's Principle. This discontinuity is a layer of concentrated vorticity as defined by Equation (1) and it is commonly called a vortex sheet. This vortex sheet is unstable and rolls up as illustrated in Figure 4.

Further insight into the lift-circulation relationship can be obtained from the modeling of the wing and its vortex wake by discrete line vortices of equal strength. Noting that line or filament vortices cannot terminate in the interior of the fluid (The Second Theorem of Helmholtz), one obtains the classical configuration illustrated in Figure 5. The vortex sheet is replaced by an arbitrary number of trailing vortex filaments. The horseshoe-shaped bound vortices lie on the surface of the wing, replacing the solid wing boundary with fluid streamlines. The starting vortex is generated when the wing accelerates from rest. It satisfies Helmholtz's Second Theorem by closing the loop of the trailing and bound vortices. Since the lift generated at any spanwise chord is given by Equation (12), the lift is directly proportional to the number of bound vortices which cut across a given chord. Placement of these bound vortices thus determines the spanwise distribution of circulation $\Gamma(y)$. The amount of vorticity in the wake at any spanwise position is given by the density of the trailing vortex lines. Mathematically this vorticity is related to the wing circulation by:

$$\gamma = \frac{d\Gamma}{dy} \tag{14}$$

where the vortex strength γ is the magnitude of the vorticity $\vec{\zeta}$ of the trailing vortex line.

The task of modeling the vortex sheet rollup has been an extremely difficult one because of the fundamental nonlinearity of the governing Equations (10) and (11). The most popular approach to the problem has been to replace the vortex sheet by a finite number of vortex filaments and to perform a discrete numerical calculation to track the filaments over time. This technique was pioneered by Westwater (1935), but was largely ignored after that until the advent of the digital computer. One of these initial computer studies was that of Moore (1971) which cast serious doubt on the numerical stability of vortex filament calculations. However, the problem was later solved by the use of finite core vortex filament models (Chorin and Bernard, 1973). A recent paper by Stremel (1984) briefly reviewed this history of vortex sheet numerical modeling and presented a 'state of the art' numerical technique. Stremel compared his numerical results with experimental data and found excellent agreement.

A fundamentally different approach to the problem of vortex sheet rollup was the use of similarity solutions as pioneered by Kaden (1931). Kaden's solution suffered from the lack of any standard of length making it difficult to relate to experimental aircraft data. As a result of this limitation, Kaden's solution has been used infrequently, although two notable studies (Westwater, 1935 and Spreiter and Sacks, 1951) have modified his similarity solution results for comparison.

There are a number of global vortex invariants which allow numerous features of the downstream tip vortices to be deduced from the aerodynamic properties of the wing, without any consideration of the rollup process. These global invariants are Γ_o, $\bar{y}$, and I_o (Widnall, 1975) and are defined by Kelvin's Circulation Theorem:

$$\Gamma_o = \int_o^{b/2} \left\{ \frac{d\Gamma(y)}{dy} \right\} dy \tag{15}$$

by Conservation of Momentum:

$$\bar{y} = \frac{1}{\Gamma_o} \int_o^{b/2} \Gamma(y)\,dy \tag{16}$$

and by the Moment of Inertia of Vortex Distribution:

$$I_o = - \int_o^{b/2} (y - \bar{y})^2 \left(\frac{d\Gamma}{dy} \right) dy \tag{17}$$

The variable Γ_o is the midspan circulation of the wing, $\bar{y}$ is the center of gravity of the semi-span vorticity, and I_o is the moment of inertia of the semi-span vorticity. These equations are only valid for two-dimensional motion in the aircraft wake (Treffetz Plane), but the invariants themselves are valid for the general three-dimensional problem.

For wings with sufficiently high aspect ratio (AR > 4) and sufficiently small leading-edge sweep angle, (< 30°), Kaden (1931) showed that the vortices are fully rolled up at a distance ℓ semi-spans behind the wing trailing edge, where ℓ is given by:

$$\ell = 0.56\ AR/C_L \tag{18}$$

where C_L is the aircraft lift coefficient. Experimentally, the vortex sheet behind the wing is observed to roll up within 20 spans (Lissaman et al, 1973, p. 2-7) into two circular vortices having opposite rotations. Therefore, it is customary to model the far-field trailing vortices as two counter-rotating, axisymmetric, finite core vortex filaments. The cores of these vortex filaments contain all of the vorticity shed from the wing, with net circulation Γ_o per semi-span as defined by (15), at locations defined by (16) (that is, centered on the center of gravity) and with a distribution satisfying (17).

With this model, and assuming elliptic loading, the following two results can be calculated (Milne-Thomson, 1952, p. 206):

$$b' = \frac{\pi}{4}\ b \tag{19}$$

$$a = 0.0855\ b \tag{20}$$

where b' is the distance between the centres of the trailing vortex filaments ('vortex spacing'), and a is the size of the vortex core.

Betz (1933) was the first to define and apply the global invariants in an attempt to ascertain the location and structure of the trailing vortices. Spreiter and Sacks (1951) used the method of Betz to develop a model for predicting the downwash due to these trailing vortex filaments. Their model stood for a long time in the face of contradictory experimental evidence until it was modified by Donaldson et al (1974). With their modified Betz theory, Donaldson et al were able to produce theoretical results that were in good agreement with experimental flight data.

The foregoing is an overview of the basic features of aircraft generated vortices. However, the subject of aircraft wakes is multi-faceted and very detailed, and no attempt will be made in this report to expand upon the basic theory presented above. Nevertheless, there is significant value in presenting a list of references pertaining to aircraft wakes that has been compiled during the writing of this report. This list can be found following the References in the Bibliography on Aircraft Wakes.

3.0 VORTEX BREAKDOWN

3.1 Introduction

Vortex breakdown was first detected by Peckham and Atkinson (1957) in their study of a highly swept delta wing at large angles of attack. Since that time, considerable effort has been expended in an attempt to understand and to predict the occurrence of vortex breakdown. These efforts have included a variety of research techniques such as analytical solutions to approximate equations of motion, numerical analysis, tube-generated vortex experiments, small scale model tests in wind tunnels and water tanks and full scale aircraft flight tests. Despite these studies, the fundamental problems of vortex breakdown have not been solved. Not only are the causes of vortex breakdown poorly understood, but the description of the phenomenon itself is uncertain.

Some features of vortex breakdown are readily identified. The process is abrupt and essentially catastrophic. A vortex filament which undergoes breakdown is destroyed or at least much diminished in strength and organization. Axial flow within the vortex core plays an important role in the process and breakdown regions typically contain stagnation points as well as reversed axial flow. Upstream of the breakdown location, the vortex filament appears unaffected; downstream of the breakdown location, the flow is disorganized and wake-like. Further structural details of vortex breakdown will be discussed later. That discussion will also review the uncertainties surrounding these details.

The discovery of vortex breakdown by Peckham and Atkinson in 1957 was quickly followed by many other experimental studies aimed at describing the phenomenon in much greater detail. Reports by Elle (1960), Werlé (1960) and Lambourne and Bryer (1961) describe much of this early work with delta wing generated vortices. The vortex breakdown phenomenon is vividly illustrated in Figure 6, a picture taken by Lambourne and Bryer (1961). The marker fluid which traces the vortex cores from the leading edge is uniform and organized until the breakdown point is reached in each vortex. This breakdown is seen to be an abrupt change which appears to have two forms: the top vortex develops a helical shape while the bottom vortex develops a bubble-like shape. Each vortex quickly degenerates into a chaotic, wake-like motion after the breakdown.

Vortex breakdown was quickly observed to occur in vortices that had not been generated by delta wings. Smith and Bessemer (1959) observed vortex breakdown in aircraft trailing vortices, and Harvey (1962) produced vortex breakdowns in solitary vortices that had been generated with his tube and vane apparatus. As a result of these two discoveries, vortex breakdown was seen to be not just a property of delta wing vortices, but a property of vortical flows in general.

At the end of 1962, the solution of the problem of vortex breakdown seemed to be well in hand. The experiments of Harvey (1962) and the theory of Benjamin (1962) were the major sources of this optimism. Both of their papers will be described in the next section. Unfortunately, some of the critical problems that were to hamper vortex breakdown research had already emerged. The most important obstacles were as follows: the extreme mathematical difficulty of the equations of motion; the complex, ill-understood mechanics of the vortex sheet rollup process behind the wing; the extreme sensitivity of the vortices to probe interference; the spatial wandering of the breakdown location; and the difficulty in assessing the effect of atmospheric conditions such as turbulence and stratification. The theoretical difficulties led to a number of solutions of approximate equations of motion whose relative merits were controversial. The experimental problems led researchers away from aircraft tests

and into tests with tube generated vortices as pioneered by Harvey (1962). These tube generated vortices were well understood and more importantly, the properties of these vortices could be changed in a carefully controlled manner which facilitated the search for critical parameters. It is important to note that the probe sensitivity of the vortices left flow visualization as the only technique available for analyzing the structure of the vortices and the breakdown process until the advent of laser doppler anemometry.

Research from 1962 onwards followed two distinct lines of investigation. The first line consisted of theoretical analyses of vortex breakdown based on various approximations to the equations of motion, supplemented by some experiments on solitary vortices generated by several forms of a tube and vane apparatus. This track of research will be discussed in Section 3.2. The second line of research did not begin until the 1970's, and it consisted of aircraft flight tests supplemented by mostly qualitative theoretical arguments. The purpose for this line of research was to obtain empirical data which would quantify aircraft wake hazards and thereby guide aircraft designers and aircraft operators. Research on vortex stability and vortex breakdown formed a part of this investigation, and it will be discussed in Section 3.3.

3.2 Vortex Breakdown Theories and Experiments

The first theoretical explanations for vortex breakdown appeared shortly after the initial discovery of the phenomenon. The theories of Jones (1960) and Ludweig (1961) were based on the idea that vortex breakdown was the result of hydrodynamic instability of the vortex core with respect to asymmetric spiral disturbances. They cited the existence of helical breakdown (Fig. 6, top vortex) as support for their theories; however, the existence of axisymmetric bubble-like breakdown (Fig. 6, bottom vortex) could not be explained by the same theories. In contrast, Squire (1960) offered an explanation based on the concept that downstream disturbances could propagate upstream and disrupt the flow in the form of vortex breakdown. This theory was subsequently reinterpreted and expanded upon by Benjamin (1962).

Benjamin's 1962 paper draws considerable support from the experiments of Harvey (1962), so it would be best to briefly describe those experiments first. Harvey produced solitary vortices in transparent tubes and used smoke for flow visualization. He observed only the bubble-like form of vortex breakdown. Furthermore, he discovered that the flow downstream of a bubble-like vortex breakdown did not have to be completely disorganized, but it could be briefly restored to that of a normal vortex under the right circumstances. Because of this, Harvey speculated that vortex breakdown was actually a transition between two kinds of organized swirling flows. This speculation was placed on a firm theoretical footing by the criticality theory of Benjamin (1962).

Benjamin rejected the idea that vortex breakdown was the result of hydrodynamic instability (Jones 1960, Ludweig 1961) on the grounds that the vortices generated by Harvey (1962) were stable and axially symmetric upstream of the breakdown location. Instead, Benjamin suggested that vortex breakdown was analogous to a hydraulic jump in that the flows upstream and downstream of the vortex breakdown were conjugate flow states. He considered the propagation of small, axisymmetric dispersive waves and discovered flow conditions for which these vortex waves could not propagate upstream (supercritical) and for which these vortex waves could propagate upstream and downstream (subcritical). He surmised that upstream of the vortex breakdown the flow was supercritical, and that downstream of the vortex breakdown the flow was subcritical. Furthermore, he demonstrated that the vital parameter was the swirl of the vortical flow, defined as the ratio of the azimuthal to the axial velocity. As the swirl of the flow increased, an initially supercritical flow would be driven towards the critical state at which location a vortex breakdown would occur. It is important to note that Benjamin's theory did not describe the details of the breakdown process, nor did it allow one to predict the location of the breakdown. Nevertheless, his theory did relate downstream and upstream flow conditions and it shed some light on the role of wave propagation of the vortex. Most importantly, Benjamin's supercritical versus subcritical classification scheme has proven to be the foundation upon which most subsequent analyses were based.

A third theory was put forward by Hall (1967). He treated the vortex prior to breakdown as quasi-cylindrical. Specifically, he assumed that the axial gradients were much smaller that the radial gradients in the vortex core. Hall developed quasi-cylindrical equations of motion and then numerically computed the downstream evolution of the vortex for a wide range of flow conditions. He found that for certain conditions, the calculation would reach a point where numerical convergence failed. He interpreted this as the vortex breakdown point which corresponded to the failure of the quasi-cylindrical assumptions. He justified this speculation by claiming it was an analogue of boundary layer separation corresponding to the failure of the boundary layer approximation. As with the theories of Benjamin, Jones and Ludweig, Hall was able to support parts of his theory by drawing upon selected tube-vortex experimental results. Specifically, his numerical experiments showed similar dependences on external pressure gradients, swirl levels, Reynold's number and cross-sectional area as did the tube-vortex experiments.

The preceding discussion of the three main vortex breakdown theories was necessarily superficial, and the reader is directed to the papers themselves for the mathematical details. It should be noted that all three theories have undergone modification over the years. A review article by Hall (1972) summarized this development and contained a superb critical analysis of the strengths and weaknesses of all three theories. An earlier review article by Hall (1966) summarized the state of knowledge on the structure and behaviour of concentrated vortices. Finally, the experiments of Kirkpatrick (1964) and Sarpkaya (1971) should be noted since they provided much of the experimental basis upon which Hall evaluated the three theories in his 1972 review.

None of the above-mentioned theories attempted to explain the details of the breakdown process. The theories only demonstrated that a vortex flow might possess certain properties such as instability or wave propagation that would allow abrupt phenomena like vortex breakdown to occur. Also, there was essentially no experimental data collected from aircraft flight tests on vortex breakdown between 1962 and 1970. Therefore, the evolution of the theories discussed in this section was guided by the tube and vane experiments of Harvey (1962), Kirkpatrick (1964) and Sarpkaya (1971), and not by aircraft experiments. The crucial question of whether or not these theories would be applicable to aircraft vortices was not seriously addressed.

The research effort on vortex breakdown after Hall (1972) was primarily devoted to increasingly sophisticated vortical wave theories and to water tank experiments on solitary tube-generated vortices. Additionally, there were some efforts at numerical simulation of the breakdown process using the full, axisymmetric Navier-Stokes equations. A pair of review articles written by Leibovich (1978, 1983) discussed and evaluated most of the research conducted after 1972. In the following paragraphs, an attempt will be made to highlight some of the significant results that were obtained.

The vortex breakdown experiments conducted during this period provided data on the structural details of the phenomenon and identified a progression of breakdown modes as a function of swirl and Reynold's number. Faler and Leibovich (1977) observed six distinctive types of vortex breakdown, some of which had been observed by Sarpkaya (1971, 1974). These breakdown forms followed each other in a repeatable order as either the swirl or the Reynold's number was increased. The bubble-like breakdown form was observed at the highest swirl and Reynold's number flows. The helical breakdown form was observed at the second highest swirl and Reynold's number flows. The other four breakdown forms resembled helical breakdown in some respects; however, little research has been done specifically on these forms because they have never been observed over delta wings or in trailing vortices.

The structural details of vortex breakdown were mapped using laser doppler anemometers in the experiments of Faler (1976), Faler and Leibovich (1977, 1978), Garg (1977) and Escudier et al (1980). The results of these experiments provided the first precise data on the axial and radial velocity profiles within the vortex core at all stages of the breakdown process. Leibovich (1983) discussed the analysis of this data in which he was able to characterize the vortices by a single parameter denoted as 'q'. On the basis of this characterization, Leibovich was able to calculate the status of the vortex at any location as being subcritical, critical or supercritical. He was able to find a reasonable agreement between the location at which the flow was calculated to be critical and the location of the vortex breakdown in the tube vortex experiments.

Most of the theoretical investigations of vortex breakdown were based on linear or non-linear wave models. Papers by Leibovich (1970), Landahl (1972), Bilanin (1973), Leibovich and Randall (1973), Randall and Leibovich (1973) and Leibovich and Stewartson (1983) describe much of this work. These models were extensions of the work begun by Squire (1960) and Benjamin (1962) and demonstrated with reasonable certainty that wave propagation within the vortex core could produce vortex breakdown under the proper conditions. Furthermore, these models were successful in describing some, but not all, of the structural features of vortex breakdown. Finally, these wave theories made significant progress towards understanding the relationship between vortex breakdown and vortex stability.

Leibovich (1978) discussed the numerical experiments of Lavan et al (1969), Kopecky and Torrance (1973) and Grabowski and Berger (1976). Leibovich concluded that these numerical experiments "convincingly" demonstrated axially symmetric solutions which resembled bubble-like vortex breakdown, and that these solutions must be unstable to non-axisymmetric disturbances. These studies continued the earlier efforts by Hall (1967) and Mager (1972) and showed that vortex breakdown could occur without wave propagation in the vortex. In fact, these numerical experiments seemed indifferent to the classifications of the criticality hypothesis. Whereas Benjamin's criticality theory requires all flows upstream of the breakdown to be supercritical, the upstream flows of Kopecky and Torrance were always subcritical, and those of Grabowski and Berger were sometimes subcritical and sometimes supercritical. The reason for this major discrepancy between the two theories is not known. Finally, it should be noted that the numerical models correctly predicted the positional response of the bubble-like breakdown location with changing upstream flow conditions, but that they did not correctly predict all of the structural details observed in the experiments.

In summary, the upgraded versions of Benjamin's criticality theory and Hall's quasi-cylindrical theory both predict the occurrence of vortex breakdown in some cases, and do so with reasonable accuracy. Neither theory has been successful at explaining all facets of vortex breakdown occurring in vortex tube experiments. It seems likely that vortex breakdown phenomena incorporate elements of both theories along with elements of the hydrodynamic instability theory of Jones (1960) and Ludweig (1961, 1965). The relative importances of the various mechanisms under the variety of possible flow conditions is one of the major questions that must be answered. However, vortex breakdown phenomena exhibit such complexity that any comprehensive model will probably have to include detailed consideration of the combined effects of asymmetrical features, nonlinearity, unsteadiness and turbulence.

3.3 Core Bursting in Aircraft Flight Tests

Recent aircraft wake research can be roughly said to start with the works of McCormick et al (1968), Crow (1970) and the conference on *Aircraft Wake Turbulence and its Detection* (1971). Since then, there have been numerous publications on the subject of aircraft wakes, a sampling of which is given in the Bibliography. The specialized topic of vortex breakdown, or vortex core bursting as it is usually called by experimental aerodynamicists, has received only a small fraction of this research effort. Vortex core bursting refers specifically to vortex breakdown in aircraft trailing vortices and it is related to a form of trailing vortex behaviour known as sinuous instability. Trailing vortices tend to deform in a sinuous fashion due to the mutual induction effects between the two vortices. Under the right circumstances, these deformations will grow until the vortices touch, at which point they will link and form vortex rings. The basic theory of sinuous instability and linking was developed by Crow (1970).

From an aeronautical viewpoint, vortex core bursting is virtually impossible to consider in isolation because of its relationships with sinuous instability, vortex wake decay and vortex wake transport. In the succeeding paragraphs, the publications of particular significance to vortex core bursting will be noted. Following that, the core bursting phenomenon will be described and then an overview of core bursting knowledge will be presented.

The most significant research effort was begun in 1970 by Meteorology Research Inc. under the sponsorship of the Air Force Office of Scientific Research, and continued by AeroVironment Inc. under the sponsorship of the Transportation Systems Center of the U.S. Department of Transportation. This research program comprised the bulk of the flight test experiments on vortex wakes in the United States and it included numerous theoretical studies directed at understanding the effect of the atmosphere on vortex wake transport, decay, stability and breakdown.

The bulk of the research program is described by the following reports: Tombach (1971), Tombach (1972), Lissaman et al (1973), Tombach (1973), Tombach et al (1974), Tombach et al (1975), Barber et al (1975), Tombach et al (1977) and Burnham et al (1978). In particular, Lissaman et al (1973) presented a broad overview of the aircraft wake vortex topic and included an extensive bibliography. Additionally, Burnham et al (1978) reported final results and discussed the findings of the entire program.

Tombach et al (1977) summarized the findings on vortex core bursting which resulted from that test program. They presented an equation which correlated the time-to-burst of a trailing vortex with the ambient atmospheric turbulence and aircraft parameters. Tombach et al did not attempt to formulate a theory which would explain why vortex core bursting occurred; instead, they employed heuristic arguments to generate their time-to-burst equation. Their equation will be presented shortly.

Three other publications which are not directly related to the major research program described above are relevant to the vortex core bursting phenomenon. The first one is a comprehensive overview of the subject of aircraft wakes written by Donaldson and Bilanin (1975). This review contained an extensive discussion of vortex breakdown mechanics based heavily on Benjamin's criticality theory and the theory of wave-trapping formulated by Landahl (1972). The review also contained a massive list of 105 references. The second paper is a review by Widnall (1975) on the mechanics of vortex filaments. Although this review was only peripherally associated with vortex core bursting, it did summarize the state of knowledge of vortex filaments and their use in modeling aircraft trailing vortices. The third paper concerns a set of water tank experiments conducted by Sarpkaya (1983). Sarpkaya towed a variety of wings through his water tank and used coloured dye for flow visualization. Some of his observations and conclusions are quite striking and will be discussed at the end of this section.

At this time, an attempt will be made to synthesize the findings of the various studies and present the state of knowledge on vortex core bursting phenomenon. It should be emphasized that the mechanics of core bursting are still not understood very well despite the extensive efforts discussed in Section 3.2. We shall start with a description of the phenomenon as observed in the flight tests and then proceed to a qualitative discussion of the experimental results and the properties of core bursting that have been deduced from these results.

Almost all of the experimental data on vortex core bursting has been obtained from visual observation of smoke-marked vortices. In the words of Tombach et al (1977): "The experiments show that the smoke-marked vortex appears to 'bunch up' and 'break'; that is, the marked core abruptly increases in diameter and then appears to burst, with the burst moving quite rapidly along the vortex axis". This phenomenon is well illustrated in Figure 7, a set of photographs taken by Sarpkaya (1983). Note that this 'core bursting' effect is apparently not the same as the spiral or bubble vortex breakdown modes observed over delta wings or in tubes. This apparent difference will be discussed at the end of this section.

Core burst frequently dissipated the marking smoke to the point of invisibility. On other occsions, a much smaller smoke-marked core remained after the burst. Anemometer measurements from Burnham et al (1978) consistently showed that the velocity profile within the vortex core was flattened by core bursting. The resulting peak core velocity was usually 30% of its former value, although in a few cases, the vortex core was completely destroyed. Furthermore, Burnham et al (1978) discovered that, "The flow visualization showed no visible difference between the partial and total vortex-decay breakdowns, except that the presence of residual visible core corresponded (obviously) to a case of partial alleviation". They concluded that the flow visualization of vortex core bursting did not provide a complete picture of the phenomenon.

Core bursting is generally thought to be a single vortex phenomenon. This has been inferred from single vortex breakdown in tubes, observations of core bursting in one vortex not affecting the other vortex, and observations of vortex bursting long after the other vortex in the pair had decayed. Nevertheless, sinuous instability always preceded core bursting, and Sarpkaya (1983) observed that breakdown occurred at points of maximum vortex separation, as had been predicted by Bilanin and Widnall (1973). The current consensus is that vortex breakdown is a local phenomenon related to the mechanics of the vortex itself and can occur in the absence of sinuous instability; however, if it is present, sinuous instability will tend to accelerate the vortex breakdown process.

It is important to emphasize that not all aircraft trailing vortices undergo core bursting. Vortex linking due to sinuous instability accounts for most of the other vortex fates, with viscous decay of the vortex occurring in a very few instances. Tombach (1973) observed that the ambient atmospheric turbulence was the key deciding factor involved. He found that core bursting occurred over the entire range of turbulence levels whereas vortex linking rarely occurred at low levels of turbulence. More recently, Sarpkaya (1983) found that vortex breakdown occurred in 30% of his sharp edged wing model tests and 10% of his rounded edged wing model tests. Sarpkaya's tests were conducted in homogeneous and density stratified water tanks for which the ambient turbulence levels were extremely low. The influence of ground effect on sinuous instability has been shown to be minimal by Tombach et al (1975), typically on the order of a 10% reduction in the required time to linking. The corresponding influence of ground effect on core bursting is not known. Nevertheless, it is interesting to note that the low altitude tests of a Boeing-747 (Burnham et al, 1978) and of a Harvard (Drummond, 1979) contained no instances of vortex linking due to sinuous instability; the vortices were observed to either core burst or persist beyond the time of observability.

The apparently probabilistic nature of core bursting has been a major obstacle to the formulation of a theory which would describe the phenomenon. Another major obstacle has been the extreme complexity of the mathematics which has thus far admitted only approximate solutions as discussed in Section 3.2. In an attempt to circumvent these difficulties, Tombach et al (1977) constructed a heuristic argument that provided an equation to correlate their time-to-burst data with aircraft properties and ambient atmospheric turbulence. The general form of their equation was:

$$T_b = G \frac{b^2}{\Gamma_o} \left(\frac{\Gamma_o^3}{(k + \epsilon) b^4} \right)^{\alpha} \tag{21}$$

where: G is a constant, b is the wing span, Γ_o is the original vortex circulation, k is the configuration constant, ϵ is the turbulence dissipation rate and α is a constant, most likely an integer multiple of 1/3.

Experimental data were analyzed for four aircraft: a Boeing-747, a Lockheed L-18 Lodestar, an Aero Commander 560F and a Cessna 170. Tombach et al (1977) found that $\alpha = 1/3$ fitted the data best, that a good choice for G was 1/3, and that good choices for k were 0.05 $cm^2 s^{-3}$ for an aircraft in clean configuration and 4.0 $cm^2 s^{-3}$ for an aircraft with flaps and gear down. Substituting the first two values into Equation (20) yields.

$$T_b \simeq \frac{1}{3} \left(\frac{b^2}{k + \epsilon} \right)^{1/3} \tag{22}$$

Tombach et al (1977) claim that this equation is accurate to within a factor of 2 for most aircraft.

The final item to be discussed in this section is a conclusion drawn by Sarpkaya (1983) based on his water tank experiments. Sarpkaya found that core bursting of the trailing vortices tended to be stationary along the axis parallel to the motion of the wing. Specifically, the growing bulge of the vortex did not translate upstream or downstream even when another core burst occurred upstream of it. In fact, vortex core bursting did not seem to have any significant effect on the rest of the trailing vortex. These observations contradicted those made by Sarpkaya (1971) during his experiments on solitary vortices generated in tubes. He concluded that, ". . . the vortex breakdown in trailing vortices does not signal a transition from supercritical to subcritical flow. It appears that what is commonly called vortex breakdown in trailing vortices is a somewhat different phenomenon and it would be more appropritate to call it 'core bulging and bursting'". This conclusion, if it proves to be valid, casts considerable doubt onto the application of the criticality theory of Benjamin to describing core bursting in aircraft trailing vortices.

4.0 CONCLUSIONS

Research into vortex breakdown phenomena has proven to be exceedingly difficult. Numerous approximate theories have been discussed in this report, as well as a variety of experimental observations. Partial successes have been achieved in modeling the breakdown phenomenon, in predicting its location in tube vortex experiments and in correlating the time-to-vortex-burst as a function of aircraft and atmospheric parameters. A comprehensive theory of vortex breakdown will probably require the combined consideration of vortex unsteadiness, asymmetry, nonlinearity and the effects of ambient turbulence in the fluid medium. An important question that will have to be answered is what are the relationship between tube-generated vortices, delta wing vortices, and aircraft trailing vortices. Until that question is answered, the vortex breakdown theories discussed in this report cannot confidently be applied to aircraft vortex breakdown.

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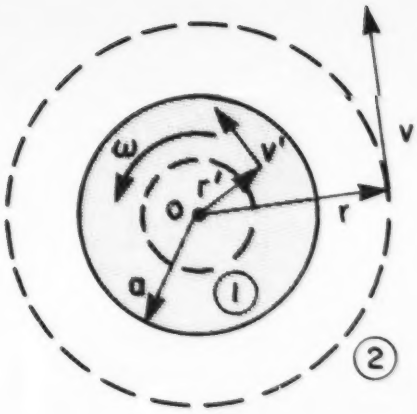

THE SHADED REGION IS THE VORTEX CORE, RADIUS a. THE VELOCITY (V OR V') IS A CONSTANT AROUND EACH CIRCULAR CONTOUR.

FIG. 1: RECTILINEAR VORTEX

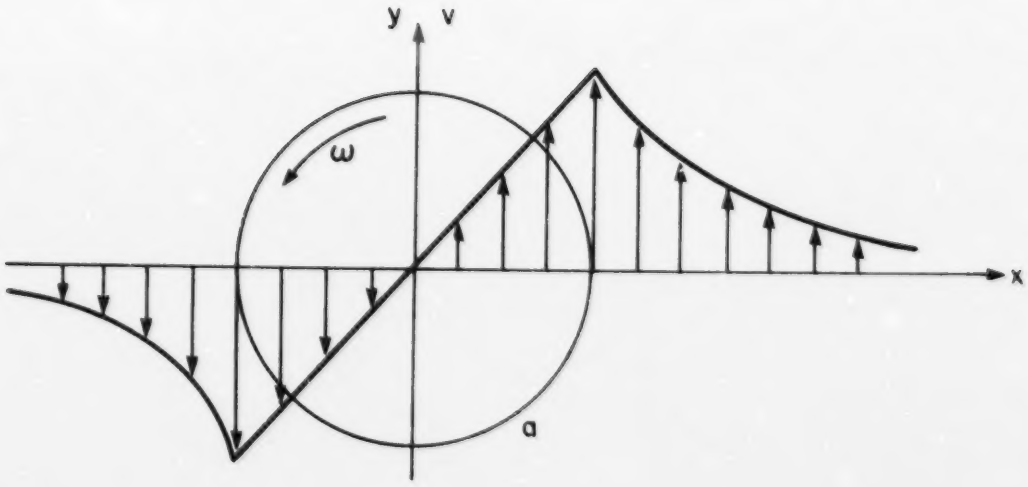

FIG. 2: INDUCED VELOCITY ALONG THE X-AXIS

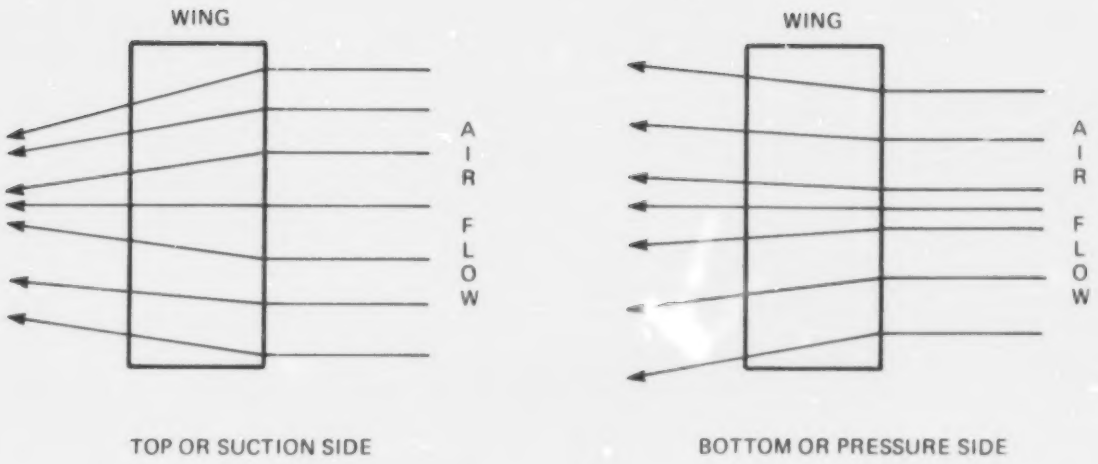

FIG. 3: SCHEMATIC DIAGRAM OF AIR FLOW PAST A WING

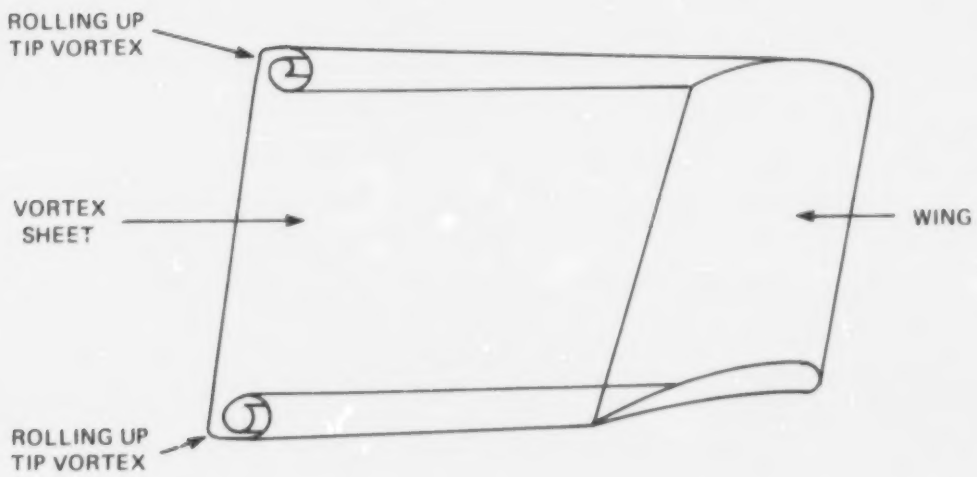

FIG. 4: ROLLUP OF TRAILING VORTEX SHEET

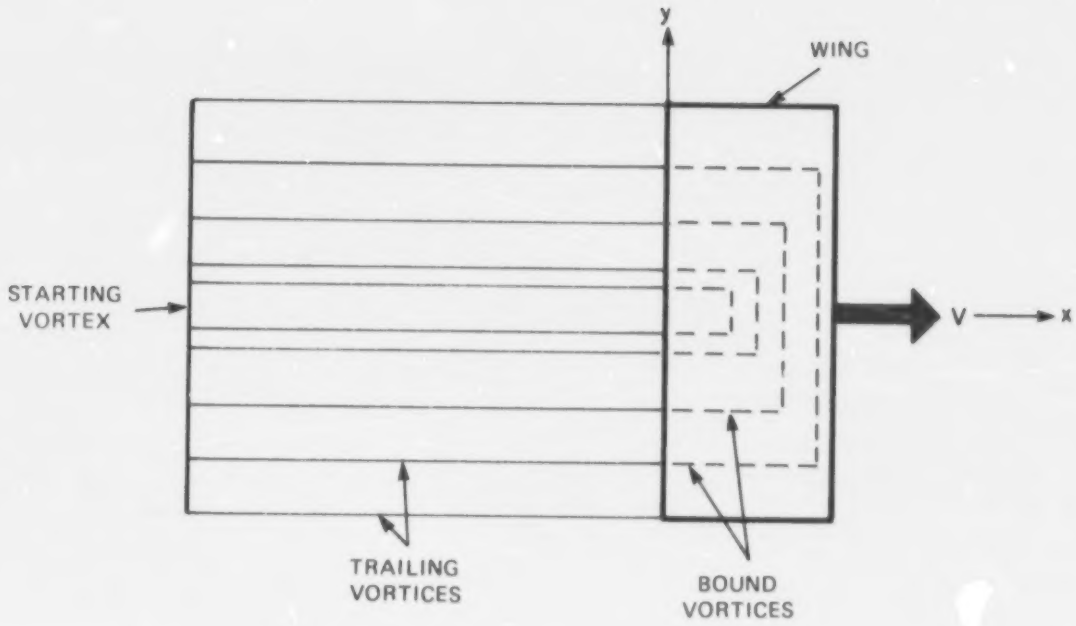

FIG. 5: DISCRETE VORTEX REPRESENTATION OF A LIFTING WING

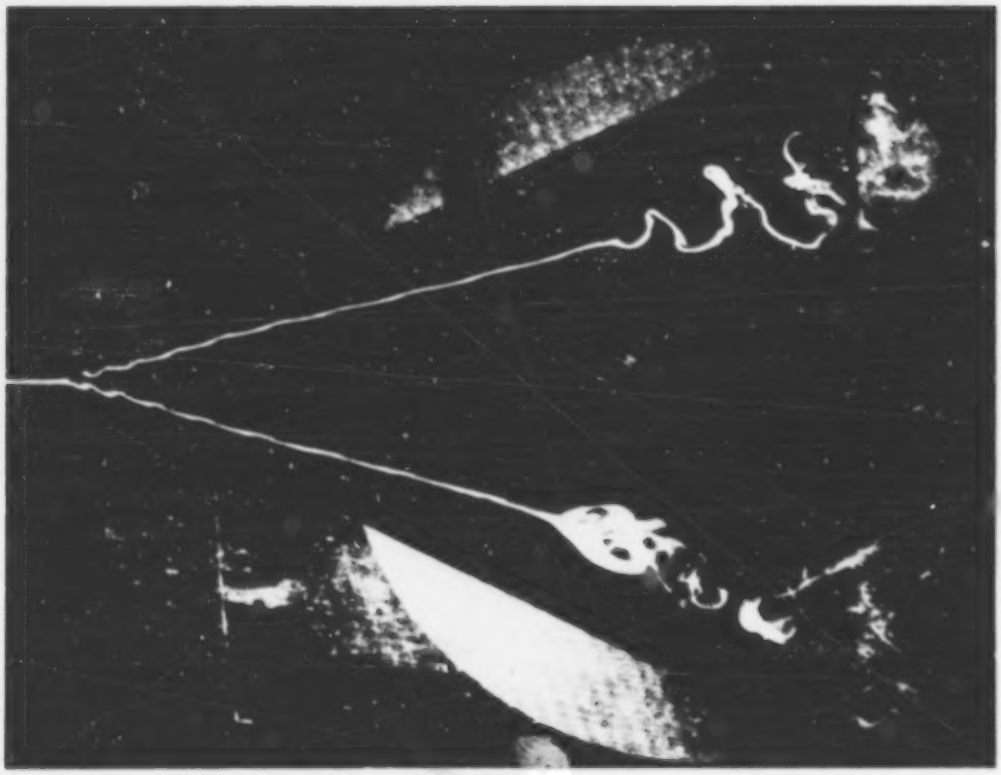

FIG. 6: VORTEX BREAKDOWN OVER A DELTA WING
(LAMBOURNE AND BRYER, 1961)

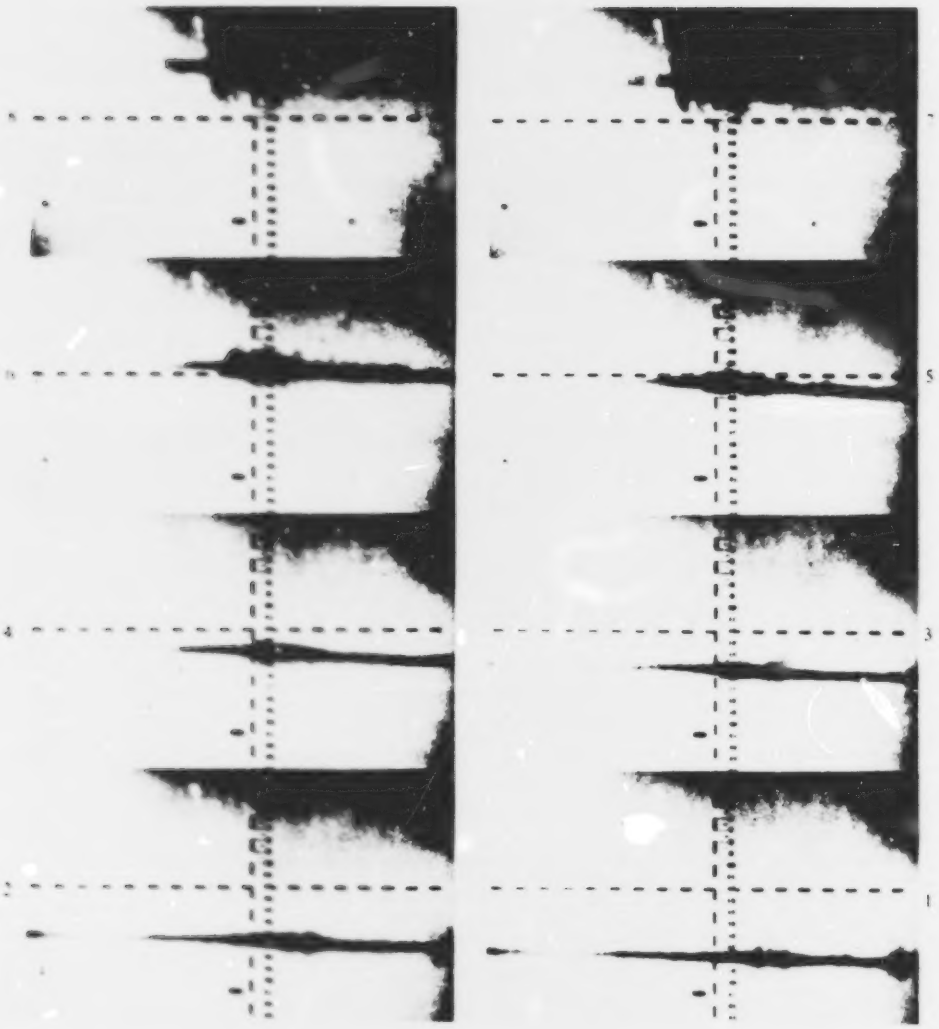

FIG. 7: EVOLUTION AND CORE BULGING AND BURSTING IN TRAILING VORTICES GENERATED BY A NACA-0012 WING (SARPKAYA, 1983)

REPORT DOCUMENTATION PAGE / PAGE DE DOCUMENTATION DE RAPPORT

REPORT/RAPPORT	REPORT/RAPPORT
NAE-AN-28 (1a)	NRC No. 24336 (1b)
REPORT SECURITY CLASSIFICATION / CLASSIFICATION DE SÉCURITÉ DE RAPPORT	DISTRIBUTION (LIMITATIONS)
Unclassified (2)	Unlimited (3)

TITLE/SUBTITLE/TITRE/SOUS-TITRE

An Introduction to Vortex Breakdown and Vortex Core Bursting (4)

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SERIES/SÉRIE

Aeronautical Note (6)

CORPORATE AUTHOR/PERFORMING AGENCY/AUTEUR D'ENTREPRISE/AGENCE D'EXÉCUTION

National Research Council Canada
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85-03 (9)	(10)	(11)	27 (12a)	7 (12b)

NOTES

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DESCRIPTORS (KEY WORDS)/MOTS-CLÉS

1. Vortex motion — theory
2. Vortex breakdown

(14)

SUMMARY/SOMMAIRE

This report is an introduction to the phenomenon of vortex breakdown, also know as vortex core bursting. The first part of the report presents some important elements of basic vortex theory. The second part reviews the research that has been done on vortex breakdown, including an overview of the historical development of vortex breakdown research and a summary of the current state of knowledge on the subject.

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